Simulating Ground Water–Lake Interactions: Approaches and Insights

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Abstract

Approaches for modeling lake-ground water interactions have evolved significantly from early simulations that used fixed lake stages specified as constant head to sophisticated LAK packages for MODFLOW. Although model input can be complex, the LAK package capabilities and output are superior to methods that rely on a fixed lake stage and compare well to other simple methods where lake stage can be calculated. Regardless of the approach, guidelines presented here for model grid size, location of three-dimensional flow, and extent of vertical capture can facilitate the construction of appropriately detailed models that simulate important lake-ground water interactions without adding unnecessary complexity. In addition to MODFLOW approaches, lake simulation has been formulated in terms of analytic elements. The analytic element lake package had acceptable agreement with a published LAK1 problem, even though there were differences in the total lake conductance and number of layers used in the two models. The grid size used in the original LAK1 problem, however, violated a grid size guideline presented in this paper. Grid sensitivity analyses demonstrated that an appreciable discrepancy in the distribution of stream and lake flux was related to the large grid size used in the original LAK1 problem. This artifact is expected regardless of MODFLOW LAK package used. When the grid size was reduced, a finite-difference formulation approached the analytic element results. These insights and guidelines can help ensure that the proper lake simulation tool is being selected and applied.

Introduction

Ground water can be important for understanding lake systems because it can influence a lake’s water budget, nutrient budget, and acid buffering capacity. As a result, ground water flows to and from lakes have often been estimated using simple flow nets and one-dimensional Darcian calculations. Ground water interaction with lakes can be spatially (Krabbenhof 1986) and temporally (Sacks et al. 1992) variable, however. Other water balance approaches have also been used (Krohelski et al. 1999) that employed a representative ground water head underneath the lake for calculating flux over the entire lake area. The most sophisticated way of investigating lake–ground water interactions is by explicitly including lakes into ground water flow models.

Although lake-ground water simulations discussed in the following used the well-known finite-difference code MODFLOW, some work used analytic element techniques. A short description of the analytic element method is given here because it is not as widely known as finite-difference techniques. Briefly, an infinite aquifer is assumed in analytic element modeling. The problem domain does not require a grid or involve interpolation between cells. To construct an analytic element model, features important to ground water flow (for example, wells) and surface water features are entered as mathematical elements or strings of elements. Each element is represented by an analytic solution. The effects of these individual solutions are superposed or added together to arrive at a solution for the ground water flow system. Because the solution is not confined to a grid, heads and flows can be computed anywhere in the model domain without nodal averaging. In the GFLOW model used here, the analytic elements are two-dimensional and are used to only simulate steady-state conditions. Therefore, the analytic element models described have application for problems where a transient solution or detailed knowledge of the three-dimensional head and flows near the lake-ground water interface are not needed. The reader is referred to Strack (1989) and Haitjema (1995) for more in-depth information about the method.
Table 1
Comparison of Techniques for Simulating Lake-Ground Water Interaction

<table>
<thead>
<tr>
<th>Advantage</th>
<th>High-Conductivity Zone</th>
<th>LAK Package</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Stable</td>
<td>• Calculates lake stage</td>
<td>• Calculates lake stage</td>
</tr>
<tr>
<td>• Simulates seepage and drainage lakes</td>
<td>• Stable</td>
<td>• Simulates seepage and drainage lakes</td>
</tr>
<tr>
<td></td>
<td>• Preprocessors available</td>
<td>• Extensive lake reporting</td>
</tr>
<tr>
<td></td>
<td>• Can simulate steady-state and transient problem</td>
<td>• Can simulate steady-state and transient problems</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Can simulate surface water flows(^1) and solute transport (LAK3)</td>
</tr>
</tbody>
</table>

| Disadvantage                                   |                                                                                   |                                                                             |
| • Cannot simulate stage                       | • Seepage lakes only                                                               | • Complex input                                                            |
| • Post-processing less mature                 | • Flux post-processing less mature                                               | • Potential instability at high conductivity                               |
| • Primarily steady-state                      | • Cannot include other surface water flows                                       | • May provide incorrect answer without user intervention                   |
| • Cannot include other surface water flows    |                                                                                   |                                                                             |
| • Infinite water source                       |                                                                                   |                                                                             |

\(^1\)LAK\(^1\) requires a modification to the STR package (Cheng and Anderson 1993) and LAK3 requires either modified version of the STR package (Merritt and Konikow 2000) or the recently developed "Stream Flow Routing" (SFR1) package (Preude et al., in review).

Regardless of the modeling code, approaches for simulating lake-ground water interactions have evolved as the tools available have grown more powerful. Early simulations employed cross-sectional models with fixed lake stages specified as specified head or head-dependent flux boundaries (Winter 1976; Anderson and Hunter 1981). Areal models were also constructed with fixed lake stages using both finite-difference and analytic element methods (Hunt et al. 1998). Other approaches were also used that allowed for lake stages to fluctuate by using a high conductivity feature or "high-K lake" (Lee 1996; Hunt and Krohelski 1996; Hunt et al. 2000). In this approach, the lake is represented by cells of high hydraulic conductivity in a finite-difference model, or as a high conductivity inhomogeneity in an analytic element model. Recently, sophisticated lake packages for MODFLOW have been developed (LAK1—Cheng and Anderson 1993; LAK2—Council 1998; LAK3—Merritt and Konikow 2000). These packages have a similar underlying approach used to solve for lake stage and calculate a lake budget; ground water flows are calculated using the lake stage and aquifer head in each cell adjacent to the lake. All have comprehensive reporting of lake water budget. The LAK packages differ primarily in features such as inclusion of a steady-state solver (LAK2, LAK3), the ability to simulate solute transport (LAK3), and inclusion of different solution schemes for solving for transient lake stage (LAK3).

When using MODFLOW, LAK packages are superior to other methods (constant head, head dependent flux, or high conductivity lake) because they have powerful post-simulation reporting features and allow for explicit inclusion of surface water flow to and from lakes. The number of publications that feature the LAK package to study ground water/surface-water interactions (Cheng and Anderson 1994; Hunt et al. 1998; Merritt 2000; Anderson et al. 2002; Krohelski et al. 2002) is still small, however. The limited use of LAK packages is attributable, at least in part, to the lack of standardization and associated graphical user interfaces, complex three-dimensional discretization and data needs, and spatial and temporal complexity inherent to including a surface water feature in a ground water model. With the advent of a public domain version of the LAK3 package, and associated incorporation into existing graphical user interfaces, we expect that this encompassing but complex methodology will become more widely used.

The objective of this paper is threefold:

1. Compare different approaches to modeling lake-ground water interactions.
2. Address the conceptualization and parameterization of lake-ground water models.
3. Introduce a new analytic element approach to lake-ground water modeling.

This paper does not provide a thorough review of all possible methods. It discusses the commonly used methods as well as selected recently developed techniques. Moreover, this work was intended primarily for the practitioner who is trying to simulate lakes in their site of interest. Accordingly, the paper is organized as follows. First, results of different lake simulation techniques are compared. Next, we present some guidelines for lake-ground water model design and conceptualization including grid size criteria, two-dimensional versus three-dimensional flow, and understanding lake capture of ground water flow. Finally, we describe an analytic element approach to lake-ground water modeling and compare its results to published LAK1 results.
Comparing Lake Simulation Techniques

Three different approaches for simulating lake-groundwater interactions (fixed lake stages, high-K nodes, and LAK3 package) are compared using GFLOW and MODFLOW models. Advantages and disadvantages of these approaches are summarized in Table 1. While sophisticated approaches offer more detail, their advantages may be offset by associated complexity and even instability of the solution procedure. Consequently, a full-featured LAK package is not an automatic choice; rather, the method chosen should depend on both hydrogeological conditions and modeling objectives.

Fixed Stage Lake Compared to a High Conductivity Zone Lake

A modified version of the Middle Genesee lakes-yeargroundwater model (Hunt et al. 2000) was used to investigate the effects of recharge reductions due to urbanization on lake levels. Middle Genesee Lake is a 44-hectare seepage lake (no surface water inlet or outlet) located in southeastern Wisconsin. The shallow ground water system consists of 120-foot-thick saturated glacial outwash. A parameter estimation code (UCODE; Poeter and Hill 1998) was coupled to an analytic element code (GFLOW; Haitjema 1995) to obtain a quantitative best-fit calibration and estimation of uncertainty around predicted lake stage decline that could result from basin development (recharge reduction). For our discussion here we will focus on differences in model predictions that resulted from two different model representations of the lakes in the area of interest. A high-K zone is used for Middle and Lower Genesee Lakes (Figure 1) in both cases, while the two upgradient lakes (immediately northeast of Middle Genesee Lake) were simulated using specified heads (fixed stage) in one case and as high-K lakes in the other case.

Results: When the two upgradient lakes were simulated using specified head conditions, the model showed a change in the Middle Genesee Lake stage between 0 and 0.12 m, as represented by the UCODE-predicted 95% confidence interval (Figure 1). When the upgradient lakes were represented as high-K zones (thus did not artificially hold

ground water levels in the area of the lake), the 95% confidence interval predicted by UCODE was in the range of 0.03 to 0.6 m. The upper boundary of the range is six times greater than for the case where the upgradient lake stage was fixed.

High Conductivity Zone Lake Compared to a LAK3 Lake

The Pretty Lake lake-groundwater model originally published by Hunt and Krohelski (1996) was recently revisited by Anderson et al. (2002) to investigate the comparability of a lake simulated using high conductivity (high-K) cells with that simulated using the LAK3 package for MODFLOW. Pretty Lake, a shallow 26-hectare, sandy-bottomed flow-through lake, is located 10 km south of Middle Genesee Lake in a similar ground water system. The objective of this study was to evaluate the solutions obtained from two different lake simulation techniques: a high-K lake and using LAK3. The problem is briefly described here; the reader is referred to Anderson et al. (2002) for a complete description. The high-K lake was input as cells with horizontal hydraulic conductivity 1000 times higher than that of the surrounding aquifer and was incised into the top four layers of a seven-layer model. Unlike the high-K lake, the LAK3 package is a customized package; therefore, there were differences in how some model parameters were input. The Horizontal Flow Barrier package (Hsieh and Freeckle 1993) was used to include the effect of lake sediments on horizontal lake-aquifer exchange in the high-K lake. For the LAK3 package, the effects of lake sediments are entered directly into the LAK3 input. Precipitation and evaporation rates were entered separately into the LAK3 package but were input as a net rate in the Recharge Package in the high-K simulation. Water added to the lake during the pumping test was defined directly in the LAK3 Package and was injected into a well placed in a cell in layer 2 of the lake in the high-K lake model. A steady-state solution served as the initial conditions for simulation of a pumping test conducted on the lake system. Although a transient calibration was not the focus of this work, simulated lake levels were compared to a time series of levels measured in the field.

Results: The same steady-state lake stage of 263.33 m was calculated by both the high-K method and LAK3, which compares well with the field-measured value of 263.29 m, and the previous analytic element modeling of Hunt and Krohelski (1996) of 263.30 m. Results from the transient simulation of the pumping test show good agreement between the high-K and LAK3 lakes, and compare fairly well with field data (Figure 2). While a transient calibration or sensitivity analysis was not performed, it was noted that the solution of the LAK3 model was more stable when lower values of lakebed leakance were used. The LAK3 steady-state lake-stage solver requires estimates of maximum and minimum lake stage to ensure convergence (Merritt and Konikow 2000). In contrast, the high-K solution was stable regardless of the value used for lakebed sediment conductivity, and did not use bounding lake stages. On the other hand, the finite-difference high-K lake can be used only to simulate seepage lakes (no surface water inflows or outflows), involves the horizontal flow barrier
Guidelines for Lake-Ground Water Models

Here we discuss two related model design issues: cell-size selection and whether to use a two-dimensional or a three-dimensional flow model. We also present a simplified conceptualization of lake-ground water interactions that can be of help in assessing the validity of more sophisticated modeling results. As shown previously, lake-ground water interactions are often subtle and sensitive to hydrogeological conditions. To successfully model these interactions, it is important to properly design the lake-ground water flow model and to critically review the results.

Cell-Size Selection Criteria for Ground Water–Lake Interaction

De Lange (1998) investigated the role of leakage between surface water features and the ground water flow regime and demonstrated that a characteristic leakage length, \( \lambda \) (units = length), is often the controlling parameter. The characteristic leakage length is defined as \( \lambda = \sqrt{cT} \), where \( T \) is the aquifer transmissivity and \( c \) is the resistance between the surface water and the ground water flow regime. The resistance \( c \) is defined as the thickness of the resistive layer (e.g., sediments) divided by the vertical conductivity of the layer. Haitjema et al. (2001) showed that for an accurate representation of the flow into or out of a surface water feature the MODFLOW grid size should not exceed \( \lambda \); ideally, the cell size should be as small as 0.1 \( \lambda \). This guideline is based on the observation that about 95% of all water enters or leaves a lake within a distance of 3 \( \lambda \) from the lake perimeter (Figure 3), which serves as the effective leakage zone. Consequently, to reasonably represent the seepage rate variations normal to the lake perimeter, at least several MODFLOW cells must be distributed over that leakage zone.

 Modeling Two- or Three-Dimensional Flow

Representing the three-dimensional geometry of a lake-ground water system requires multiple grid layers to properly include the vertical dimension of the problem. In view of limited computer resources, both in terms of RAM and processor power, a detailed three-dimensional model places constraints on the allowable grid resolution. Yet, as seen in the previous section, cell sizes near and underneath the lake should be on the order of the characteristic leakage length \( \lambda \) or smaller in order to properly represent the ground water flow field. Depending on the aquifer and aquitard properties, a single layer model may be best suited to satisfy this cell-size constraint. The question then is can a single layer (Dupuit-Forchheimer) model adequately represent the three-dimensional ground water flow regime?

Three-dimensional flow can be represented in Dupuit-Forchheimer models, but resistance to vertical flow is ignored (Kirkham 1967; Strack 1984). A Dupuit-Forchheimer model of lake-ground water interactions may form a good approximation to a fully three-dimensional model when the lake is large compared to the aquifer thickness. Haitjema (1987) showed that for a circular infiltration area with a diameter of about five times the aquifer thickness, the differences between the approximate and exact (three-dimensional) potentiometric head surfaces and ground water pathlines become very small. These findings, however, apply to isotropic aquifers. An anisotropic aquifer may be replaced by an isotropic aquifer with a thickness that is \( \sqrt{k_h/k_v} \) times the actual aquifer thickness, where \( k_h \) and \( k_v \) are the horizontal and vertical hydraulic conductivity of the aquifer, respectively. Defining a characteristic length \( L \) as \( 5\sqrt{k_h/k_v} \), times the saturated aquifer thickness may lead, based on the findings of Haitjema (1987), to the following guideline: Dupuit-Forchheimer models may be used to model lake-ground water interactions as long as the length and width of the lake are not less than the characteristic length \( L \) and as long as the distances between impor-
tant hydrologic features such as rivers, streams, and wetlands are not less than L.

Vertical Lake Capture in Ground Water-Lake Interactions

A common and useful way to evaluate a model's construction is to perform a simple hand calculation using Darcy's law, generalized aquifer properties, and observed potentiometric gradients surrounding the lake. However, while the gradient may indicate flow toward or away from the lake, it is not certain that flow over the entire aquifer height enters the lake or originates from the lake. The lake may in reality skim off water flowing in the upper part of the aquifer and release water that does not fully penetrate to the aquifer bottom. While representative gradient and hydraulic conductivity estimates can be uncertain, this partial penetration, if it exists, also complicates the use of Darcy's law in verifying lake inflows and outflows in the model. Some insight into whether or not the lake impacts the ground water regime over its full depth may be obtained from work by Townley et al. (1993), and other works (Townley and Davidson 1988; Nield et al. 1994; Townley and Trefry 2000; Smith and Townley 2002).

Simple concepts important to lake capture developed by Townley and Davidson (1988) are illustrated in Figure 4. Figures 4b to 4d were generated using FLOWTHRU (Townley et al. 1992). FLOWTHRU displays a two-dimensional vertical section through water bodies that are long in the direction perpendicular to the direction of regional ground water flow. The code uses a set of precomputed solutions obtained using a linear triangular finite-element model that encompass a variety of lake configurations, aquifer parameters, and boundary conditions. The code illustrates lake-ground water flow and flux distributions through a cross section of a lake-aquifer system. The visualization includes the option to display equipotentials and flowlines, and thus can serve as a rough check of input entered into more sophisticated lake-ground water models.

The simple conceptualizations illustrated in Figure 4 are possible because in many cases "lakes are windows into the water table. Their influence on nearby ground water flow systems comes from the fact that the resistance to flow within the water body itself is much less than in the ground" (Townley, written communication, 2001). Analysis of flow-through lakes in aquifer systems otherwise dominated by two-dimensional flow shows that the magnitude of the "short-circuiting" effect of the lake in such settings is controlled largely by the depth of the lake, the ratio of the lake diameter to aquifer thickness, and the vertical anisotropy of the system. Deep lakes that penetrate most of the aquifer give rise to two-dimensional flow systems. The fully penetrating capture zone of such a lake, shown in plan view, extends more than twice its diameter, A, along a line perpendicular to the direction of regional aquifer flow (Figure 4a).
Table 2
Model Description and Parameters Used in the LAK1, Analytic Element Lake, and Grid Sensitivity Runs

<table>
<thead>
<tr>
<th></th>
<th>LAK1</th>
<th>Analytic Element Lake</th>
<th>Grid Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layers</td>
<td>2</td>
<td>1</td>
<td>1 and 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_s/K_a$</td>
<td>100:1</td>
<td>1:1</td>
<td>1:1 (1 layer)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lake simulated using</td>
<td>MODFLOW LAK1 package</td>
<td>Analytic element lake</td>
<td>MODFLOW constant head cells</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lake level solved for?</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grid spacing</td>
<td>152 m × 152 m</td>
<td>NA</td>
<td>152 m × 152 m to 3 m × 3 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of cells</td>
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<td>416 to 1,040,000</td>
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<td></td>
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</tr>
<tr>
<td>Aquifer $K_h$</td>
<td>61 m/day</td>
<td>61 m/day</td>
<td>61 m/day</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lake $K_b$</td>
<td>NA</td>
<td>NA</td>
<td>122 m/day</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recharge</td>
<td>29.1 cm/yr</td>
<td>29.1 cm/yr</td>
<td>29.1 cm/yr</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stream stage</td>
<td>Specified; obtained from</td>
<td>Specified; obtained from</td>
<td></td>
</tr>
<tr>
<td>Solved for by MODFLOW stream (STR) package (Pitulie 1989)</td>
<td>LAK1 output</td>
<td>LAK1 output</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stream width</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stream sediment $K_s$</td>
<td>0.04 m/day</td>
<td>0.04 m/day</td>
<td>0.04 m/day</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stream sediment thickness</td>
<td>0.3 m</td>
<td>0.3 m</td>
<td>0.3 m</td>
</tr>
</tbody>
</table>

NA = Not applicable

Shallow lakes (that is, much less deep than the aquifer thickness, B) can produce three-dimensional flow systems where the lake does not fully capture the flow over the whole thickness of aquifer. In section view the depth of the capture zone depends on the ratio of A:B (lake diameter: aquifer thickness). Small shallow lakes (A:B = 1) tend to capture half the aquifer thickness (Figure 4b); large shallow lakes (A ≥ 8B) tend to capture flow through the entire aquifer thickness (Figure 4c), and therefore behave like deeper lakes. However, vertical resistance to flow can modify this outcome. Large shallow lakes in an isotropic aquifer mimic the partial capture zone of a small shallow lake when there are lake sediments that have a lower hydraulic conductivity ($K_{sed}$) than the hydraulic conductivity of the aquifer ($K_s$; Figure 4d). Similar to the case with lake sediments, a large shallow lake without lake sediment in an anisotropic aquifer can also have partial penetration similar to a small shallow lake (not shown).

It should be noted that the simple seepage lake problems discussed here were constructed assuming precipitation falling on the lake equaled the amount of water evaporated off the lake, and that aquifer recharge near the lake was negligible compared to the amount of water flowing from the problem boundaries. Moreover, surface water inflow and outflow and subsurface heterogeneity were not considered. Sites that deviate from these simple problems should be investigated on a case-by-case basis.

Analytic Element Lake Package

The regional analytic element (AE) ground water flow model GFLOW (Haitjema 1995) supports stream networks with steady-state streamflow calculations. The AE lake package is a new element that is based on an enhancement of this feature and includes steady-state simulations of lakes with both ground water and surface water inflows and outflows and an initially unknown lake stage. Linesinks are used to model ground water inflow or outflow along the lake perimeter (Appendix A). Resistance to inflow will depend on lake bottom resistance (defined previously). The linesink functions incorporate this resistance and an associated width parameter (representative leakage zone near the lake perimeter) to obtain the ground water inflow and outflow rates in response to lake stage and ground water elevations. The width parameter must be chosen in such a manner that the linesink inflow or outflow corresponds to the lake inflow or outflow for the case where the entire lake bottom is represented explicitly by a resistance layer. For most cases where the lake width is large (larger than 10λ), it is appropriate to choose a width parameter equal to the characteristic leakage length λ. For explanation, and a description of width selection for atypical lake geometries, see the discussion in the Appendix.

If the lake is part of a stream network, surface water inflow from an upgradient stream and surface water outflow through an outlet stream are included. The surface water outflow is dependent on the lake stage, using a linear interpolation between a table of lake stages and associated outflow rates. Similarly, volumes of water resulting from precipitation falling on the lake and lake evaporation will depend on the surface area of the lake, which in turn depends on the lake stage. The lake surface area is obtained through a linear interpolation between a table of lake stages and associated lake surface areas. Because GFLOW is a
steady-state model, only steady-state lake stages can be simulated.

Solutions are obtained in an iterative manner as follows: The user specifies a lower and an upper estimate of the lake stage, $\varphi_1$ and $\varphi_2$, respectively. A complete conjunctive surface water and ground water solution is generated for each of the two lake stages, which result in the lake water balance deficiencies $\Delta Q_1$ and $\Delta Q_2$, respectively. A new lake stage $\varphi$ is calculated by use of the secant method:

$$\varphi = \varphi_1 - \frac{(\varphi_2 - \varphi_1) \Delta Q_1}{\Delta Q_2 - \Delta Q_1}$$

(1)

The water balance deficiency for the lake is calculated from

$$\Delta Q = \sum_i s_i l_i + \sum Q_{i} + \sum Q_{i_{in}} - \sum Q_{out} + A(\varphi)E$$

(2)

where $s_i$ is the sink density of the $i$th linesink, $l_i$ is the length of the $i$th linesink, $Q_i$ is the overland flow into the $i$th linesink, $\sum Q_{i_{in}}$ is the inflow of all inlet streams, $\sum Q_{out}$ is the outflow into all outlet streams, and $A(\varphi)E$ is the stage-dependent lake area times the net precipitation rate ($= $ precipitation rate $- $ evapotranspiration rate). Depending on the complexity of the conjunctive surface water and ground water flow problem, it may take five or more iterations (lake level calculations) to arrive at a reasonable water balance for the lake ($\Delta Q < 1\%$ of the total lake inflow or outflow). It was found that small variations in lake stage (on the order of 0.1 foot) may result in appreciable water balance errors. In other words, while the solution procedure rapidly approaches the final lake stage, a solution accurate enough to produce an accurate water budget tends to require additional iterations. Consequently, in order to consider a solution converged, one must look at changes in the lake water budgets as well as changes in lake stage.

### Analytic Element Lake Example

Using Original LAK1 Problem

As a mutual check we compared simulation results from the AE lake package to the LAK1 example problem of Cheng and Anderson (1993). The problem consists of a stream-lake system in an anisotropic aquifer bounded by no-flow boundaries (Figure 5). Parameters for the MODFLOW and AE models are given in Table 2. While there are obvious elements in the problem design (two layers, aquifer anisotropy) that preclude the construction of exactly parallel formulations, there are also more subtle differences in the simulation of the lake. The original LAK1 problem contains lakebed sediments with low conductivity sediments on the lake bottom; however, the problem design incised the lake through more than one-half the aquifer thickness and did not include low conductivity lake sediment along the vertical lake-aquifer interface. This design removed much of the effect of the low conductivity bottom sediments from the problem by facilitating exchange of water along horizontal pathways. The AE lake element does not simulate a lake-side and lake-bottom sediment component but approximated the LAK1 design by removing all resistance between the lake element and the surrounding aquifer.

The AE lake results had the same lake stage and a similar total water budget as the LAK1 results (Table 3). The most notable difference in Table 3 is how the ground water flux is distributed between the lake and adjoining streams; the differences were 21% and 24% for simulation of the outlet and inlet streamflow, respectively, and about 3% for simulated lake fluxes. While differences in problem design are present, an artifact might result from the grid size used in the original LAK1 problem. That is, the 152 m (500 foot) grid size used in the LAK1 problem violates the recommended grid size presented earlier in this paper.
Using the problem parameters in Table 2, the calculated $\lambda$ for the streams would be 83.5 m (274 feet); the cell size should be at least 1$\lambda$, with higher grid resolution (i.e., cell size equal to 0.1$\lambda$) preferable. In most applications, multiple lambda values will apply for the different streams and lakes in a given area of interest. However, given the design of the original LAK1 problem (one lake in a closed domain, bottom resistance from the bottom lake sediments having a small role in the exchange of water between the lake and aquifer), we have restricted our discussion to the lambda calculation for the inlet/outlet streams in this analysis.

A grid sensitivity analysis was performed on a modification of the LAK1 problem. Rather than using the LAK1 package, a simplified version of the problem was constructed to closely match the dynamics of the one-layer AE model. The problem used MODFLOW river cells (RIV package) for the streams and constant head cells for the lake, with lake and stream stages specified according to the LAK1 results (Table 2). Constant head cells were used to represent the lake in order to simulate the high degree of lake-aquifer connectedness included in the Cheng and Anderson (1993) LAK1 problem design. Hydraulic conductivity specified for lake constant head cells was twice that used in the aquifer to account for the constant head lake formulation using one cell width to calculate intercell conductance; LAK1 uses one-half the cell width (Cheng and Anderson 1993). The grid size was reduced in increments that allowed for the original problem geometry to remain unaffected; thus, the following grid sizes were tested: 152 m, 76 m, 38 m, 15 m, and 3 m (1.8$\lambda$, 0.9$\lambda$, 0.45$\lambda$, 0.18$\lambda$, and 0.035$\lambda$, respectively).

The grid size can have appreciable effects on simulation of distribution of flow in this lake-ground water interaction (Figure 6). Using the 152 m grid size as the original LAK1 problem, a constant head lake using a two-layer model is similar to the original LAK1 result; the 152 m result for the one-layer constant head lake is slightly different due to different total lake conductance and the absence of the anisotropy specified in the LAK1 two-layer problem. However, comparisons of the one-layer constant head lake results show that the difference introduced by the coarse grid is three times that introduced by collapsing the two-layer problem to one-layer (Figure 6), even with the anisotropy specified for the two-layer problem.

Using the 152 m and 3 m one-layer grid spacing results, the grid size artifact is estimated to be on the order of 11% and 25% for simulation of the outlet and inlet streamflow, respectively, and around 3% for simulated lake fluxes. The one-layer constant head lake results approached those obtained by the AE lake as the grid becomes more refined (Figure 6). It should be noted that the simulated fluxes are somewhat constrained by the original LAK1 problem design. That is, the problem is characterized by a constant, fixed amount of water infiltrated into a closed domain dominated by a large sink (the lake) that has a high degree of hydraulic connection to the aquifer. Water balance sensitivities may be greater in modeling applications that include multiple competing sinks of the same relative importance.

It is clear that accurate simulation of the distribution of flow between the lake and a stream could be important if the objective is to simulate the streamflow (for example, in the case where a gaging station flux target is located upstream of a lake). However, one might wonder if concerns about grid size are important when the artifacts associated with grid size affect simulated lake fluxes on the order of 3%. While the overall lake water balance is not grossly in error, it remains true that the inflow into the lake occurs essentially within 3$\lambda$ from the lakeshore, and flowlines will be affected by how accurately this near-shore environment is simulated. A coarse grid (cell size greater than $\lambda$) cannot accurately represent the distribution of the flux into the lake. Thus, if the simulation of the flux distribution to the lake is important to the modeling objective (for example, simulating where a contaminant plum might enter a lake), cell sizes smaller than $\lambda$ are indeed important.

Summary and Conclusions

Six main conclusions were reached during this study:

- How lakes are included in a ground water flow model (specified head, stage solved for by model) can have an appreciable effect on the model results. Lakes surrounding the lake of interest should also be considered for more sophisticated lake simulation techniques because fixing their stages can have unintended constraint and influence on the lake of interest.
- High-K lakes give similar results to more sophisticated lake simulation techniques (e.g., LAK3) and are generally more stable and simpler to input. However, most can be used only on seepage lakes (no surface water inlets or outlets) at the present time, can require many iterations for a good solution, and at the present time require more complex post-processing of model results to obtain lake fluxes.
- Solutions for simple lake-ground water interactions have become more extensively developed, and a tool is available on the World Wide Web (see Townley et al.)
1992) that can visualize conceptual ground water flow patterns beneath shallow lakes. Generally, a flow-through seepage lake in an isotropic aquifer that has sufficient ground water exchange (to be worth including in a ground water model) will have the following characteristics: (1) in plan view, it will capture ground water from a width twice the lake width in the direction perpendicular to ground water flow; (2) vertically, it will capture ground water from one-half the aquifer thickness if it is small (that is, if the lake length is of the order of the aquifer thickness); (3) it will capture ground water of the full aquifer thickness if it is large (that is, lake length is equal to or greater than eight times the aquifer thickness). More complex lake systems (surface water inlets and outlets, precipitation on lake appreciably different than lake evaporation) will deviate from these broad generalizations.

- Defining a characteristic length \( L \) as \( 5 \sqrt{\frac{k_h}{k_v}} \), where \( T \) is the aquifer transmissivity (underneath the lake) and \( c \) the resistance to flow between the lake and the underlying aquifer. The resistance \( c \) is defined as the thickness of the resistance layer (e.g., sediments) divided by the vertical hydraulic conductivity of the layer. About 95% of all water enters or leaves a lake within a distance of 3\( \lambda \) from the shore. Thus, multiple cells are needed to accurately simulate the flux distribution. Ideally, the cell size used in a model should be on the order of 0.1\( \lambda \). However, cell sizes on the order of \( \lambda \) are often acceptable.

- An analytic element lake package has been formulated that is comparable to the LAK packages for MODFLOW, but is limited to two-dimensional steady-state flow. The lake can be isolated or part of a stream network, with surface water inflow from an upgradient stream and surface water outflow through an outlet stream. The surface water outflow is dependent on the lake stage, using a linear interpolation between a table of lake stages and associated outflow rates. The analytic element lake calculated the same lake stage, and simulated flow to the lake differed by about 3% from a published LAK1 model. Differences in results are mostly attributed to the large cell size of the LAK1 problem, as determined performing grid sensitivity on a modified one-layer version of the LAK1 problem. The largest discrepancy between the coarse and fine grid was a 25% difference in the simulation of the inlet stream flux. The finite-difference formulation approached the analytic element fluxes as the grid size was reduced.

Approaches for modeling lake-ground water interactions have evolved over time, including the development of a lake “feature” for analytic element modeling. Despite (and in some case because of) increasing sophistication, the potential for “mismodeling” has also increased. With clearly stated modeling objectives, by avoiding undue model complexity, and by comparing modeling results to basic conceptual solutions to lake-ground water interactions, the modeler can ensure that the correct tool is being selected and properly applied.

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Authors’ Note

Any use of trade, product, or firm names is for descriptive purposes only and does not imply endorsement by the U.S. government.

References


![Figure A1. Conceptual model of ground water flow into a lake.](image)

**Appendix:** Selecting the Line-Sink Width Parameter

Resistance to flow through a low permeable lake bottom is conceptualized in Figure A1 as follows. The lake bottom has a thickness $d$ and a hydraulic conductivity $k_b$, which leads to a resistance $c$ [days] defined as

$$c = \frac{d}{k_b} \quad (A1)$$

The flow underneath the lake bottom is considered Dupuit-Forchheimer flow, which means that resistance to vertical flow inside the aquifer is ignored. Conversely, the flow through the lake bottom is one-dimensional vertical flow, ignoring any horizontal flow in the lake bottom. Horizontal flow into the sides of the lake is ignored as well. The aquifer underneath the lake has a thickness $H$ and hydraulic conductivity $k$. The head in the aquifer at the lake boundary is $\phi_0$, while the water level in the lake is $\phi$, both measured with respect to the aquifer bottom. The head $\phi(x)$ underneath the lake bottom is given by (Verruijt 1970, page 30, Equation 4.8)

$$\phi = \phi_0 + \alpha e^{x/\lambda} + \beta e^{-x/\lambda} \quad (A2)$$

where $\alpha$ and $\beta$ are integration constants to be determined from boundary conditions. The coordinate direction $x$ is perpendicular to the lake boundary, with $x = 0$ at the lake boundary, while the parameter $\lambda$ [m] is the "characteristic leakage length" defined as
\[ \lambda = \sqrt{\frac{kHc}{\rho}} \]  

(A3)

Further on in this appendix we will show that 95\% of the leakage into the lake occurs within a distance of 3\( \lambda \) from the shore. We will assume that \( \lambda \) is small compared to the dimensions of the lake. Under these circumstances the leakage through the lake bottom occurs principally near the lake boundary as defined by line-sinks used to simulate the flow into the lake. Each line-sink is withdrawing water from the aquifer at a rate \( \sigma \) [m\(^2\)/day] (m\(^3\)/day per meter length of the line-sink) that follows from

\[ \sigma = \frac{\phi_0 - \phi_s}{c} w \]  

(A4)

where \( w \) [m] is a width parameter associated with the line-sink when resistance to flow into a surface water feature is to be included.

To ensure that the line-sinks along the lake boundary are withdrawing the correct amount of water, the conceptual model of flow into the line-sinks is compared to that of flow into the lake (O.D.L. Strack, personal communication, April 2000). The ground water flow rate \( Q_x \) [m\(^3\)/day] underneath the lake, perpendicular to the lake boundary, is obtained by differentiating Equation A2:

\[ Q_x = -kH \frac{\partial \phi}{\partial x} = -\frac{kH}{\lambda} (\alpha e^{x/\lambda} - \beta e^{-x/\lambda}) \]  

(A5)

Since the leakage through the lake bottom decreases at an increasing distance from the lake boundary, \( Q_x \) will vanish for large values of \( x \). Theoretically, at \( x = \infty \) the head in the aquifer will have become equal to the lake level and no further leakage occurs into the lake, thus \( Q_x \) becomes zero at \( x = \infty \). From this it follows that

\[ \alpha = 0 \]  

(A6)

The head in the aquifer at the lake boundary \( (x = 0) \) is \( \phi_0 \), which leads with Equation A2 to

\[ \beta = \phi_0 - \phi_s \]  

(A7)

The flow rate \( Q_x \) in the aquifer at the lake boundary \( (x = 0) \) must equal the inflow rate \( \sigma \) into the line-sink that is positioned at that boundary. Hence, with Equations A4 through A7 and \( x = 0 \) it follows that

\[ \frac{\phi_0 - \phi_s}{c} w = \frac{kH (\phi_0 - \phi_s)}{\lambda} \]  

(A8)

which leads to the following expression for \( w \):

\[ w = \frac{\lambda}{\lambda} \]  

(A9)

In other words, the line-sink width \( w \) must be selected equal to the characteristic leakage length \( \lambda \) to obtain the proper inflow rate into the line-sinks representing the lake. This analysis, of course, also holds true when the lake loses water to the aquifer.

As stated earlier, 95\% of the leakage into the lake occurs over a distance of 3\( \lambda \) from the lake boundary, that is in the domain \( 0 < x < 3 \lambda \), which can be shown as follows. The total leakage into the lake between \( x = 3 \lambda \) and \( x = \infty \), becomes (with Equations A5 through A7)

\[ \frac{kH (\phi_0 - \phi_s)}{\lambda} (e^{-x} - e^{-3\lambda}) = 0.0498 \frac{kH (\phi_0 - \phi_s)}{\lambda} \]  

(A10)

which is 5\% of the total lake inflow as given by Equation A8. Consequently, 95\% of the flow enters the lake between \( x = 0 \) and \( x = 3 \lambda \), or within a distance of 3\( \lambda \) from the shore.

Narrow Elongated Lakes

In some instances a lake may be narrow and elongated, which could compromise the assumption that the distance \( B \) between opposite lake boundaries is large compared to the characteristic leakage length \( \lambda \). The lake then resembles a wide stream with line-sinks along each shore. The analysis presented here can be adapted by assuming that \( Q_x = 0 \) midway on the lake (\( x = B/2 \)). This leads to the following rules for selecting the line-sink width \( w \) (Haitjema 2001):

\[ w = \lambda \]  

\[ \lambda \leq 0.1B \]  

(A11)

\[ w = \lambda \tanh(B/2\lambda) \]  

\[ 0.1B < \lambda < 2B \]  

(A12)

\[ w = B/2 \]  

\[ \lambda \geq 2B \]  

(A13)

The analysis presented here is based on one-dimensional flow perpendicular to the lake boundary, which is an approximation of the actual flow patterns near the lake. Similarly, opposite lake shores will rarely be truly parallel as suggested by the conceptual model in Figure A1. Consequently, the lake width \( B \) is not precisely defined, but merely is an estimate to decide on the use of Equation A11, A12, or A13 for determining the appropriate line-sink width \( w \).